II Semester B.A./B.Sc. Examination, May 2017
(CBCS) (F + R) (2014-15 and Onwards)
MATHEMATICS – II

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions : **(5×2=10)**

a) Prove that the binary operation $*$ defined by $a * b = \frac{ab}{4}$ on the set of rationals is commutative and associative.

b) Prove that the group $(Z_6, \oplus_6)$ is abelian.

c) Find the angle between the radius vector and the tangent to the curve $r = a(1 + \cos\theta)$.

d) Write the formula for radius of curvature for the curve $y = f(x)$.

e) Find the asymptotes parallel to coordinate axes to the curve $a^2y^2 + b^2x^2 = x^2y^2$.

f) Find the length of the curve $4y^2 = x^3$ between $x = 0$ and $x = 5$.

g) Show that $(ax + hy + g)dx + (hx + by + f)dy = 0$ is exact.

h) Solve : $p^2 - 5p + 6 = 0$, where $p = \frac{dy}{dx}$.

PART – B

Answer **one full** question : **(1×15=15)**

2. a) Show that the set of all fourth roots of unity forms an abelian group under multiplication.

b) Prove that a non-empty subset H of a group $(G, *)$ is a subgroup of G, if and only if

i) $a*b \in H, \forall a, b \in H$ ii) $a^{-1} \in H, \forall a \in H$.

c) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, then show that $f \circ g$ is the identity and find $(f^{-1} \circ g^{-1})$.

OR

P.T.O.

3. a) Prove that the set of complex numbers $\mathbb{C}$ is an abelian group under addition.

b) Show that $H = \{0, 2, 4\}$ is a subgroup of the group $(Z_6, \oplus_6)$.

c) If $f = \begin{pmatrix} a & b & c & d \\ b & a & d & c \end{pmatrix}$ and $g = \begin{pmatrix} a & b & c & d \\ c & d & a & b \end{pmatrix}$, then find $(f \circ g)^{-1}$ and $f^{-1} \circ g^{-1}$.

PART – C

Answer **two full** questions : **(2×15=30)**

4. a) With usual notation prove that $\tan\phi = r\frac{d\theta}{dr}$.

b) Find the angle between the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.

c) Show that evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid.

OR

5. a) Find the angle between the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$.

b) Derive the coordinates of the centre of curvature of the curve $y = f(x)$.

c) Find the pedal equation of the curve $r^n = a^n \cos n\theta$.

6. a) Find all the asymptotes to the curve $x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 - 2 = 0$.

b) Find the envelope to the curve $\frac{x}{a} + \frac{y}{b} = 1$ and $a + b = c$, where c is a parameter.

c) Find the surface area generated by the revolution of an arc of the catenary $y = c\cosh\left(\frac{x}{c}\right)$ about x-axis between $x = 0$ to $x = a$.

OR

7. a) Find the envelope of the family of lines $x\cos^3\alpha + y\sin^3\alpha = a$, where α is a parameter.

b) Determine the position and nature of the double points on the curve $x^3 - y^3 + 4y - 7x^2 + 15x - 13 = 0$.

c) Find the volume of the solid generated by the revolution of an arc of the catenary $y = c\cosh\left(\frac{x}{c}\right)$ about x-axis between $x = -a$ and $x = a$.

PART – D

Answer **one full** question : **(1×15=15)**

8. a) Solve : $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$.

   b) Solve : $p = \tan\left(x - \frac{p}{1-p^2}\right)$.

   c) Find the orthogonal trajectories to the curve $r = a(1 - \cos\theta)$.

OR

9. a) Solve : $x\frac{dy}{dx} + (1-x)y = x^2y^2$.

   b) Solve : $y = px + \sin^{-1}p$.

   c) Find the orthogonal trajectories to the curve $x^{2/3} + y^{2/3} = a^{2/3}, a > 0$.
